Note on Magnetic Field Models in Visualis

Ampèrian Model vs Gilbert 2-Poles Approximation

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1 Context

Two classical approaches exist to model the static magnetic field of permanent magnets : the *Ampèrian model* (bound currents) and the *Gilbert model* (magnetic charges).
In Visualis we use the Ampèrian Model for accuracy, and on WebGL the field is computed on GPU by a faster closed-form two-pole approximation derived from the Gilbert picture.

2 The Ampèrian Model

2.1 Principle

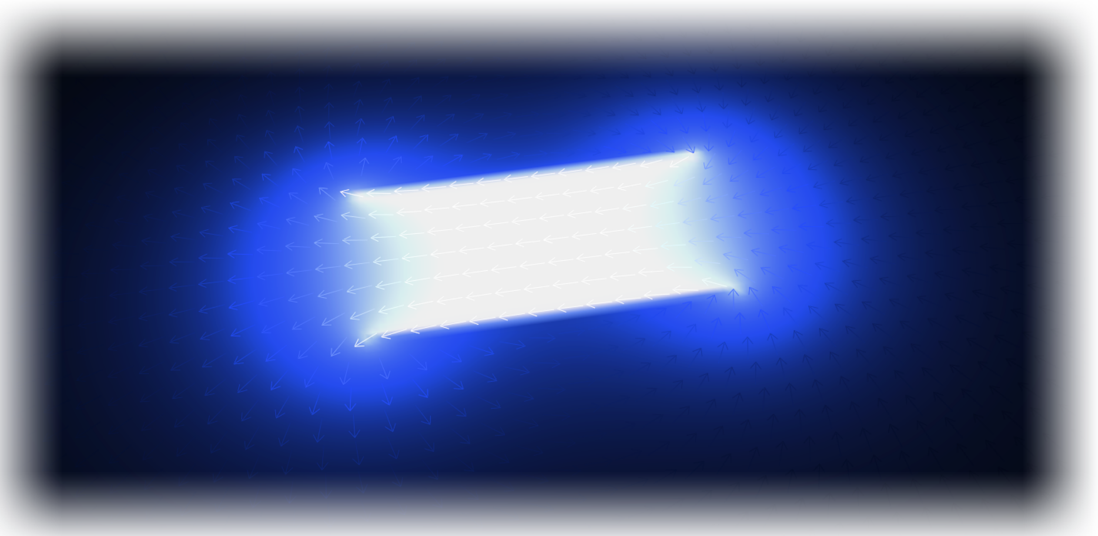
The Ampèrian (or current-loop) model represents magnetisation $\boldsymbol{M}$ as a distribution of bound currents :

$$\boldsymbol{J}_b = \nabla \times \boldsymbol{M} \quad \text{(volume bound current)} \tag{1}$$

$$\boldsymbol{K}_b = \boldsymbol{M} \times \hat{\boldsymbol{n}} \quad \text{(surface bound current)} \tag{2}$$

For a uniformly magnetised bar, $\boldsymbol{J}_b = 0$ inside, and $\boldsymbol{K}_b$ flows as a surface current on the lateral faces. The magnetic field is then obtained via the Biot–Savart law :

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{K}_b(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, \mathrm{d}S' \tag{3}$$



2.2 Computational cost

Evaluating (3) requires a numerical surface integral (or a sum over discrete current loops), making it significantly more expensive per pixel than the two-point sum of (7). For N current elements, the cost per pixel is $O(N)$ vs. $O(2)$ for the Gilbert dipole.

3 The Gilbert Two-Pole Approximation

3.1 Principle

The Gilbert model treats a bar magnet as a pair of fictitious magnetic monopoles : a positive charge $+q_m$ (North pole) and a negative charge $-q_m$ (South pole) separated by a distance ℓ.
Each monopole produces a radial field analogous to an electric point charge :

$$\boldsymbol{B}_{\text{pole}}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \frac{q_m}{|\boldsymbol{r} - \boldsymbol{r}_0|^3} (\boldsymbol{r} - \boldsymbol{r}_0) \tag{4}$$

where $\boldsymbol{r}_0$ is the pole position and q_m is the magnetic pole strength in units of A·m.

3.2 Dipole superposition

A bar magnet with half-length h and orientation $\hat{\boldsymbol{n}}$ is modelled by placing :

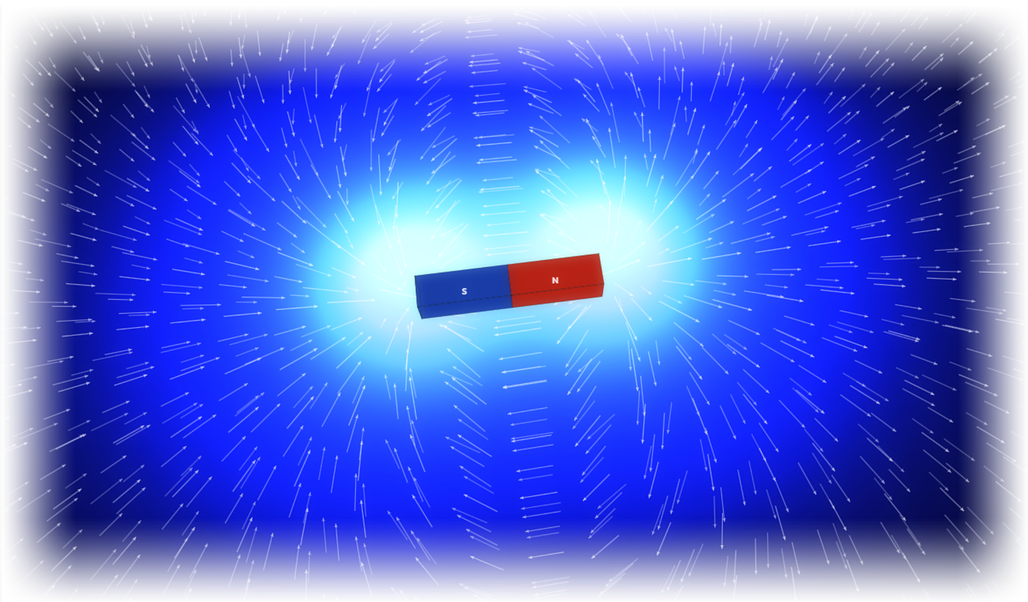
$$\text{North pole :} \quad \boldsymbol{r}_N = \boldsymbol{r}_c + h\,\hat{\boldsymbol{n}}, \quad \text{strength } +q_m \tag{5}$$

$$\text{South pole :} \quad \boldsymbol{r}_S = \boldsymbol{r}_c - h\,\hat{\boldsymbol{n}}, \quad \text{strength } -q_m \tag{6}$$

The total field is the linear superposition :

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \left[\frac{q_m\,(\boldsymbol{r} - \boldsymbol{r}_N)}{|\boldsymbol{r} - \boldsymbol{r}_N|^3} - \frac{q_m\,(\boldsymbol{r} - \boldsymbol{r}_S)}{|\boldsymbol{r} - \boldsymbol{r}_S|^3} \right] \tag{7}$$

The magnetic dipole moment is $\boldsymbol{m} = q_m \ell\, \hat{\boldsymbol{n}}$ (units : A·m^2), as the standard formulation.



3.3 Implementation in GLSL

Fragment shader of the dipole computation :

```
// Internal units (mu_0/4pi = 1)
vec3 B = vec3(0.0);
for (int i = 0; i < numPoles; i++)
{
    vec3 r = pos - polePos[i];
    float r3 = max(dot(r,r) * length(r), 5e-4);
    B += poleStr[i] * r / r3;
}
// Display: B_physical = B_internal * 1e-7 [T]
```

Note : we omit the physical constant $\mu_0/4\pi$ and work in rescaled internal units, to keep the numerical/float range convenient. The physical scaling can then be restored at display time if needed.

3.4 Physically acceptable ?

A single isolated magnetic monopole would violate Maxwell's equation $\nabla \cdot \boldsymbol{B} = 0$ (Gauss's law for magnetism), and indeed (I think ?) no magnetic monopole has ever been observed experimentally.

However, the *pair* of monopoles $(+q_m, -q_m)$ taken together produces a field that satisfies $\nabla \cdot \boldsymbol{B} = 0$ everywhere except at the two point singularities. In the far field (distance $r \gg \ell$), the Gilbert dipole is almost identical to the Ampèrian dipole :

$$\boldsymbol{B}_{\text{dipole}}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\boldsymbol{m}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{m}}{r^3}\right] \tag{8}$$

The two-point Gilbert approximation differs from the exact finite-magnet field in the near field (especially close to the poles, edges, and magnet surface) but reproduces the correct dipolar far field.

4 Summary: Model Comparison

Property	Ampèrian (currents)	Gilbert (monopoles)
Formula	$\int \boldsymbol{K} \times \boldsymbol{r}/r^3\,\mathrm{d}S$	$q_m\,\boldsymbol{r}/r^3$ (per pole)
Cost per pixel	$O(N)$ per magnet	$O(2)$ per magnet
Far-field accuracy	Exact	$\approx$ Exact
Near-field accuracy	Exact	Approximate
$\nabla \cdot \boldsymbol{B} = 0$	Everywhere, yes	At singularity, no
GPU-friendly	Expensive	**Yes**

So we choose Gilbert model for the GPU visualisation, as the main goal is realtime interactions, the two-pole superposition shows dipolar field structure at negligible computational cost ; while the Ampèrian model remains available in Visualis C++ application, to keep physical correctness of $\nabla \cdot \boldsymbol{B}$ and accuracy inside the magnet body.

https://www.visualis-physics.com/WebGL/magnetic_field_demo_2D.html
https://www.visualis-physics.com/WebGL/magnetic_field_demo_3D.html
https://www.visualis-physics.com