# PARTICLE TRAJECTORY IN UNIFORM MAGNETIC FIELD

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## Introduction

In this example, we can observe the trajectory of charged particles in a uniform magnetic field. The motion of charged particles is a fundamental concept in electromagnetism and has important applications in many areas of physics.

## Lorentz Force

The force experienced by a charged particle moving in a magnetic field is given by the Lorentz law, which states :

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

where $q$ is the charge of the particle, $\vec{E}$ is the electric field, $\vec{v}$ is the velocity of the particle, and $\vec{B}$ is the magnetic field. In a uniform magnetic field with no electric field, this simplifies to :

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2}$$

## Helical Trajectory

In a uniform magnetic field, a charged particle will move in a helical trajectory. The magnetic force on a moving particle is perpendicular to the direction of its velocity and to the direction of the magnetic field : if the particle is launched with a component of velocity perpendicular to the magnetic field, it will experience a magnetic force and its path will curve.

So we have two special cases :

- If a charged particle is moving parallel to the magnetic field, then the force on it will be zero and its path will be a straight line.
- If the charged particle is moving in the plane perpendicular to the magnetic field it will follow a circular path.

Any situation in-between will result in a helical path : the component of velocity parallel to the field remains unchanged, while the magnetic force causes circular motion in the perpendicular plane.

The radius $r$ of the helix is given by :

$$r = \frac{mv_{\perp}}{|q|B} \tag{3}$$

where $m$ is the mass of the particle, $v_{\perp}$ the component of the velocity perpendicular to the magnetic field, and $B$ the magnitude of the magnetic field.

The pitch $p$ (distance between turns) of the helix is given by :

$$p = v_{\parallel} T \tag{4}$$

where $v_{\parallel}$ is the component of the velocity parallel to the magnetic field and $T$ is the period of the circular motion, given by :

$$T = \frac{2\pi m}{|q|B} \tag{5}$$

We note that the equation for the period doesn't include the velocity of the particle. This might seem counterintuitive at first, but it makes sense when we consider the physics involved.

The equation for the period of the circular motion of a charged particle in a magnetic field is derived from the balance between the centripetal force, which is provided by the magnetic force, and the inertia of the particle.

The magnetic force on a charged particle moving in a magnetic field is given by $F = qvB$, where $q$ is the charge, $v$ is the speed of the particle, and $B$ is the magnetic field strength. This force provides the centripetal force for the circular motion, which is given by $F = mv^2/r$, where $m$ is the mass of the particle, $v$ is its speed, and $r$ is the radius of the circle.

Setting these two expressions for the force equal to each other gives :

$$qvB = \frac{mv^2}{r}$$

Solving for $r$ gives the radius of the circle :

$$r = \frac{mv}{|q|B}$$

The speed of the particle is the circumference of the circle divided by the period, or $v = 2\pi r/T$. Substituting the expression for $r$ into this equation gives :

$$v = \frac{2\pi mv}{|q|BT}$$

Solving for $T$ gives the period of the motion :

$$T = \frac{2\pi m}{|q|B}$$

So we can see that the speed of the particle actually cancels out in this derivation. The period of the motion only depends on the charge and mass of the particle and the strength of the magnetic field, not on the speed of the particle. This is a unique feature of the motion of charged particles in a magnetic field.

## Additional Notes

Since the force is always perpendicular to the direction of motion, a magnetic field cannot change the velocity of a particle. If the magnetic field is non-uniform, it is possible to control the particle's trajectory. If a particle is initially (locally) moving in the direction of the magnetic field, it will continue to do so (as long as the change in the field along its path is gradual).