PARTICLE TRAJECTORY IN UNIFORM MAGNETIC FIELD

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Introduction

In this example, we can observe the trajectory of charged particles in a uniform magnetic field. The motion of charged particles is a fundamental concept in electromagnetism and has important applications in many areas of physics.

Lorentz Force

The force experienced by a charged particle moving in a magnetic field is given by the Lorentz law, which states :

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1}$$

where q is the charge of the particle, $\vec{E}$ is the electric field, $\vec{v}$ is the velocity of the particle, and $\vec{B}$ is the magnetic field. In a uniform magnetic field with no electric field, this simplifies to :

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2}$$

Helical Trajectory

In a uniform magnetic field, a charged particle will move in a helical trajectory. The magnetic force on a moving particle is perpendicular to the direction of its velocity and to the direction of the magnetic field : if the particle is launched with a component of velocity perpendicular to the magnetic field, it will experience a magnetic force and its path will curve.

So we can consider two special cases :

- If a charged particle is moving parallel to the magnetic field, then the force on it will be zero and its path will be a straight line. Test in current scene !
- If the charged particle is moving in the plane perpendicular to the magnetic field it will follow a circular path. Test in current scene !

Any situation in-between will result in a helical path : the component of velocity parallel to the field remains unchanged, while the magnetic force causes circular motion in the perpendicular plane.

The radius r of the helix is given by :

$$r = \frac{mv_{\perp}}{|q|B} \tag{3}$$

where m is the mass of the particle, $v_{\perp}$ the component of the velocity perpendicular to the magnetic field, and B the magnitude of the magnetic field. The pitch p (distance between turns) of the helix is given by :

$$p = v_{\parallel}T \tag{4}$$

where $v_{\parallel}$ is the component of the velocity parallel to the magnetic field and T is the period of the circular motion, given by :

$$T = \frac{2\pi m}{|q|B} \tag{5}$$

we note that period only depends on the charge and mass of the particle, and the strength of the magnetic field, not on particle's velocity. Test in Scene !

Additional Notes

Since the force is always perpendicular to the direction of motion, a magnetic field cannot change the velocity of a particle. If the magnetic field is non-uniform, it is possible to control the particle's trajectory. If a particle is initially (locally) moving in the direction of the magnetic field, it will continue to do so (as long as the change in the field along its path is gradual).