CHARGE DISTRIBUTION IN A PLANE CAPACITOR

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Introduction

In this example, we can observe the distribution of electric charges in a theoretical plane capacitor. While one might initially expect the charges to distribute uniformly across the plates, the distribution is actually more complex, with a higher concentration of charges along the edges of the plates. This phenomena is known as the edge effect.

Click here to enable the simulation and see the new charges repartition !

Click here to return to initial state and compare !

Charge Distribution

In a plane capacitor, the electric field is ideally uniform and directed from the positively charged plate to the negatively charged plate. This would suggest a uniform distribution of charges on the plates. However the charges tend to accumulate more at the edges due to the mutual repulsion between like charges. (Note that with this distribution the field is actually more uniform between the plates).

Electric Field

In the ideal case of an infinite plane capacitor, the electric field between the plates is given by:

$$E = \frac{\sigma}{\varepsilon_0} \tag{1}$$

where σ is the surface charge density on the plates and ε_0 is the permittivity of free space. This field is uniform and directed from the positively charged plate to the negatively charged plate.

However, for a real finite plane capacitor, calculating the electric field at an arbitrary point in space involves integrating over the charge distribution on the plates. The electric field $d\vec{E}$ at a point due to a small element of charge dq is given by Coulomb's law:

$$d\vec{E} = \frac{kdq}{r^2}\hat{r} \tag{2}$$

where k is Coulomb's constant, r is the distance from the charge element to the point, and $\hat{r}$ is a unit vector pointing from the charge element to the point. To find the total electric field at the point, we need to integrate this expression over the charge distribution, this would typically involve a double integral over the area of the plates. (Note : for an uniform charges distribution, surprisingly this integral can be solved !)

Equipotential Surfaces

The non-uniform charge distribution affects the shape of the equipotential surfaces between the plates. Due to the edge effect, the equipotentials are more parallel to each other in the central region of the capacitor. The accumulation of charges at the edges of the plates creates a stronger electric field, which counteracts the tendency of the electric field to curve in the space between the plates. This results in a more uniform electric field, and thus more parallel equipotential surfaces.