PARTICLE TRAJECTORY IN A MAGNETIC FIELD

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Abstract. This document tries to give an overview of the behaviour of charged particles in a magnetic field, and the study of particles in Visualis Electromagnetism.

Lorentz Force and Circular Motion

The Lorentz force acting on a charged particle moving in a magnetic field is given by $\vec{F} = q(\vec{v} \times \vec{B})$, where q is the charge, $\vec{v}$ is the velocity vector of the particle, and $\vec{B}$ is the magnetic field vector. For motion perpendicular to the field ($\vec{v} \perp \vec{B}$), the force is centripetal, causing circular motion with a radius r given by $r = \frac{mv}{qB}$, where m is the mass of the particle and v is its speed.

Helical Paths

The helical trajectory of a charged particle with components of velocity both parallel and perpendicular to the magnetic field can be described mathematically using cylindrical coordinates and parametric equations. The radius of the helix is given by the expression in the previous point, while the pitch h of the helix is $h = v_{\parallel} \times T$, where $v_{\parallel}$ is the component of the velocity parallel to the magnetic field and $T = \frac{2\pi m}{qB}$ is the period of the circular motion.

No Work Done by Magnetic Field

This property is represented by the equation $W = \int \vec{F} \cdot d\vec{r} = \int (q\vec{v} \times \vec{B}) \cdot d\vec{r} = q \int \vec{v} \cdot (d\vec{r} \times \vec{B}) = 0$, since the cross product $\vec{v} \times \vec{B}$ is always orthogonal to $d\vec{r}$.

Charged Particles Curve Direction

Charged particles of opposite sign curve in opposite directions. This is a direct consequence of the force equation $\vec{F} = q(\vec{v} \times \vec{B})$, since reversing the sign of the charge q will reverse the direction of the force.

Cyclotron and Synchrotron Motion

The frequency of revolution of a charged particle in a cyclotron or synchrotron is given by the cyclotron frequency $f = \frac{qB}{2\pi m}$. In a cyclotron, this frequency matches the frequency of an applied oscillating electric field. In a synchrotron, the frequency of the electric field is adjusted as the particle speeds up to ensure it remains in phase with the oscillation.

Magnetic Mirrors

The mirror effect can be derived from the conservation of the magnetic moment $\mu = \frac{mv_{\perp}^2}{2B}$ of the charged particle, where $v_{\perp}$ is the component of the velocity perpendicular to the magnetic field. As B increases, $v_{\perp}$ decreases to keep μ constant, until the particle's direction of motion is reversed.

Visualis

All these behaviours are represented in Visualis Electromagnetism through sample scenes. The tore scene in particular shows how the particles indirectly tend to follow magnetic field lines inside a large curved electric magnet !