MAGNETIC FIELD LINES PROPERTIES

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Magnetic Field Line Loop

This property can be mathematically described using Gauss's Law for Magnetism, which states that the magnetic flux through a closed surface is zero. Here are mathematical hints :

Gauss's law for magnetism is written in integral form as follows:

$$\oint \mathbf{B} \cdot d\mathbf{A} = 0 \tag{1}$$

This equation states that the integral of the magnetic field B over a closed surface (the left-hand side of the equation) is equal to zero.

The dot product $B \cdot dA$ gives the component of the magnetic field that passes perpendicularly through a tiny area element dA on the surface. The integral sums up these contributions over the entire closed surface.

Because we're working in three dimensions, we can consider a tiny volume element dV in the shape of a cube. The contribution to the integral from this cube is:

$$\oint_{\text{cube}} \mathbf{B} \cdot d\mathbf{A} = B_x \Delta y \Delta z - B_x \Delta y \Delta z + B_y \Delta x \Delta z - B_y \Delta x \Delta z + B_z \Delta x \Delta y - B_z \Delta x \Delta y = 0 \tag{2}$$

Where B_x, B_y, and B_z are the x, y, and z components of the magnetic field. The positive terms represent magnetic field exiting the volume, and the negative terms represent magnetic field entering the volume. The sum of these is zero because the net magnetic flux through any closed surface is zero.

We can extend this argument to any arbitrary closed surface by breaking it up into tiny volume elements like this, and summing up the contributions from all of them. Therefore, the integral of the magnetic field over any closed surface is zero, which means that the magnetic field lines which are following the field gradient must always form closed loops.

Note that this assumes that magnetic monopoles do not exist. If magnetic monopoles can be found, Gauss's law for magnetism would have to be modified, and magnetic field lines would not necessarily form closed loops ; they will behave like electric field lines.

In Visualis Electromagnetism you can see how magnetic field lines come back on themselves even in complex configurations, and you can try to create a magnetic monopole thank to Dirac String hypothesis !